Research paper

Form of nonequilibrium statistical operator, thermodynamic flows and entropy production

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Nonequilibrium statistical operator (NSO) in the form of Zubarev is recorded as averaging of quasiequilibrium statistical operator on the distribution of the lifetime of the system. Form of density function of the lifetime of the system affects of all its non-equilibrium characteristics. In general, we consider the situation when the density of distribution of the lifetime of the system depends on the current time. In the expressions for the fluxes and entropy production obtained additional terms in comparison with the expressions derived from Zubarev's NSO. We view the values of these supplements for the distribution of the lifetime of the system, obtained by the principle of maximum entropy.

Keywords: non-equilibrium statistical operator, density of lifetime distribution, entropy production.

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1. Introduction

One of the most fruitful and successful ways of development of the description of the non-equilibrium phenomena is served by a method of the non-equilibrium statistical operator (*NSO*) [1, 2, 3]. In work [4] new interpretation of a method of the *NSO* is given, in which operation of taking of invariant part [1, 2, 3] or use auxiliary «weight function» (in terminology [5, 6]) in *NSO* are treated as averaging of quasi-equilibrium statistical operator on distribution of past lifetime of system. This approach is consistent with the conducted Zubarev [2] obtaining *NSO* by averaging over the initial time.

This interpretation of *NSO* gives him physical sense of the account of causality and allocation of a real final time interval in which there is a given physical system. New interpretation leads to various directions of development of *NSO* method which is compared, for example, with Prigogine's [7] approach, introduction of the operator of internal time, irreversibility at microscopically level.

In [5] source in the Liouville equation enters in the modified Liouville operator and coincides with the form of Liouville equation suggested by Prigogine [7] (the Boltzmann-Prigogine symmetry), when the irreversibility is entered in the theory on the microscopic level. We note that the form of *NSO* by Zubarev in the interpretation of [4] corresponds to the main idea of [7] in which one sets to the distribution function ρ_q which evolves according to the classical mechanics laws, the coarse distribution function $\rho = \Lambda\rho_q$ (Λ is operator) whose evolution is described probabilistically since one perform an averaging with the probability density $p_q(u)$, Λ acts as an integral operator.

In Kirkwood's works [8] it was noticed, that the system state in time present situation depends on all previous evolution of the non-equilibrium processes developing in the system. In [5, 6] it is specified, that it is possible to use many «weight functions». Any form of density of lifetime distribution gives a chance to write down a source of general view in dynamic Liouville equation which thus becomes, specified Boltzmann and Prigogine [5, 6, 7], and contains dissipative items.

If in Zubarev's works [1-3] the linear form of a source corresponding limiting exponential distribution for lifetime is used other expressions for density of lifetime distribution are giving fuller and exact analogues of "integrals of collisions». Explicit account of violation of time symmetry (a finite lifetime, its beginning and end) is introduced.

In work [9-10] it is shown, in what consequences for non-equilibrium properties of system results change of lifetime distribution of system for systems with final lifetime. In [9-11] the various dependence of the probability density of time past life $p_q(u)$ from the age of the system are considered, $u=t-t_0$, t is current time, t_0 is the moment of the birth of the system. In [11] also $p_q(u,t)$ as dependence on the current time is considered. In [11] this dependence is chosen piecewise continuous form, where in one time slot value $p_q(u)$ has a single species, in the other – the other. Perhaps we have the more general situation when the function $p_q(u,t)$ is continuous in the current time of the argument t. This case is considered in this paper – in general and for specific task of function $p_q(u,t)$. We show the effect of this function on the physical characteristics of the system: flows and entropy production.

2. New interpretation of NSO.

In [1-3] the logarithm of the Nonequilibrium Statistical Operator is introduced through operation of taking an invariant part of the operators $F_m(x,t+t_1)P_m(x,t_1)$ concerning evolution with Hamilton function H, i.e. transition from the logarithm of quasi-equilibrium statistical distribution (in a terminology [2]) $ln\rho_q(z;t+t',t')$ (z is the point in phase space describing a state of system at a microscopic mechanical level) to the logarithm of NSO

$$ln\rho(z;t)=\varepsilon\int_{-\infty}^{0} dt' exp\{\varepsilon t'\} ln\rho_q(z;t+t',t')dt', \quad (1)$$

where

$$ln\rho_q(z;t_1,t_2)=-\Phi(t_1)-\sum_{j=1}^{n} \int dx F_j(x,t_1)P_j(z|x,t_2); \quad (2)$$

$$P_0(z;x)=H(z;x);\ P_1(z;x)=p(z;x);\ P_{i+1}(z;x)=n_i(z;x); \quad (3)$$

$$F_0(x,t)=\beta(x,t);\ \ F_1(x,t)=-\beta(x,t)v(x,t);\ \ F_{i+1}(x,t)=-\beta(x,t)(\mu_i(x,t)-m_i v^2(x,t)/2);$$

$$\Phi(t)=ln\int dz exp\{-\sum_{j=1}^{n} \int dx F_j(x,t)P_j(z|x)\}. \quad (4)$$

Here $H(z;x)$ is dynamic variable density of energy, $n_i(z;x)$ are density of particles for the i-th component, $p(z;x)$ is pulse density; $\beta(x,t)$ plays a role of local reverse temperature, $\mu(x,t)$ is the local chemical potential, $v(x,t)$ is the mass speed, m_i are masses of i-th particles, x are the spatial coordinates (here pulses also can be included). The second argument t_2 in $\rho_q(t_1,t_2)$ designates dependence on time through Heizenberg representation for dynamic variable, on which the function $\rho_q(t,0)$ can explicitly depend. Other choice of the variable $P_m(z|t)$ is also possible (see for example [5]). The choice of the variables (3) corresponds to local-equilibrium distribution [1-2] (for the description of a hydrodynamic stage of a nonequilibrium process). The parameters

$F_n(t)$ are chosen so that true average of starting set of values P_n were equal to their quasi-equilibrium average

$$<P_n>^t=<P_n>^t_q=Sp(\rho_q(t)P_n). \qquad (5)$$

Thus the local integrals of motion are chosen in a retarded form, which in [1] is related to causality conditions in a formal problem of the scattering theory and theory of Bogoliubov's quasi-averages. From the complete group of solutions of Liouville equation (symmetric in time) the subset of retarded "unilateral" in time solutions is selected by means of introducing a source in the Liouville equation

$$\partial\rho/\partial t+iL\rho(t)=-\varepsilon(\rho(t)-\rho_q(t,0)), \qquad (6)$$

which tends to zero (value $\varepsilon\to 0$) after thermodynamic limiting transition. Here L is Liouville operator; $iL=-\{H,\rho\}=\Sigma_k\{(\partial H/\partial p_k)(\partial\rho/\partial q_k)-(\partial H/\partial q_k)(\partial\rho/\partial p_k)\}$; H is Hamilton function, p_k and q_k are pulses and coordinates of particles; $\{\ldots\}$ is Poisson bracket.

In [2] the operation of taking an invariant part (1) is provided by physical meaning of averaging on initial condition. The assumption is made that the evolution of system with equal probability can begin from any initial condition $\rho(t)_{|t=to}=P\rho(t_0)=\rho_q(t_0,0)$ (where P is projection operator [2]) in an interval from t_0 up to t, which is considered large enough (that became an insignificant detail of an initial condition, as dependence on the initial moment of time t_0 is nonphysical). The state $\rho(t)$ observable in the moment t is equal to an average on the initial moments of time t_0 from the solution of an initial value problem for the statistical operator (or function of distribution) $exp\{-iL(t-t_0)\}\rho_q(t_0,0)$ for an enough large interval of time $t-t_0$, necessary for damping the initial nonphysical states. In [1-2] the equality of integrals in Abel's and in Cezaro's sense is used, and the relation for *NESO* average on initial states is rewritten as

$$\rho(t)=\int_{-\infty}^{t}(1/<\Gamma>)exp\{-(t-t_0)/<\Gamma>\}exp\{-iL(t-t_0)\}\rho_q(t_0,0)dt_0. \qquad (7)$$

It is possible to assume that the system evolves as isolated from the state $\rho_q(t_0,0)$ making random transitions with exponent probability $w(t,t_0)=(1/<\Gamma>)exp\{-(t-t_0)/<\Gamma>\}$ (where $<\Gamma>=1/\varepsilon\to\infty$ after $V\to\infty$) and to interpret it as influence of a "thermostat" [2]. In [5] functions $w(t,t_0)$ are considered in a general view. The properties of these "weight functions" [5] are investigated. Other (very many) choices of the weight function w are possible [5].

In [4, 9-11] other interpretation of functions $w(t,t')$, operation (1) and *NSO* is given. Having done in (7) the replacement $t_1=t_0-t$, rewrite (7) as

$$\rho(t)=\int_{-\infty}^{0}(1/<\Gamma>)exp\{t_1/<\Gamma>\}exp\{iLt_1\}\rho_q(t+t_1,0)dt_1=$$

$$\int_{0}^{\infty}(1/<\Gamma>)exp\{-y/<\Gamma>\}exp\{-iyL\}\rho_q(t-y,0)dy, \qquad (8)$$

where in the second equality (8) arguments t_1 is replaced by $-y$. Thus $t_0-t=-y;\ t=t_0+y$, i.e. the current time t is represented as the sum of the initial moment t_0 and value y, which represents lifetime of system (random value). The Liouville operator (in a classical case) affects on dynamic variable P_n leaving values of temporary arguments in F_n equal $t_0=t-y$. The expression (8) corresponds to averaging of value $exp\{-iyL\}\rho_q(t-y,0)$ with probability density $(1/<\Gamma>)exp\{-y/<\Gamma>\}$. Last value coincides with the distribution density function of time of the first exit from set of states (lifetime) in a limiting case for one class ergodic states in the circuit of state lumping of complex systems [16] (see also [17]). In the renewal theory [18] or in the theory of reliability this value corresponds to the density of probability of non-failure operation. In the ratio (8) the value $<\Gamma>=\varepsilon^{-1}$ are interpreted as the average lifetime of the finite nonequilibrium system, i.e. the average lifetime of the random values P_j at the description of the system by the nonequilibrium distribution (1).

Except for exponential density of probability (10), as density of distribution of lifetime the Erlang distributions (special or general) for n classes of ergodic states (so, for $n=2$, $p_q(y)=\theta\rho_1 exp\{-\rho_1 y\}+(1-\theta)\rho_2 exp\{-\rho_2 y\}$) can be used, gamma-distributions etc. (see [19-20]), and also amendment taking into account subsequent terms of asymptotic expansion series [16, 17]. The value $(1/<\Gamma>)exp\{-y/<\Gamma>\}$ in expression (8) we shall replace by $p_q(y)$ - density of probability of system lifetime (or time of non-failure operation).

Thus the lifetime enters in *NSO* describing any physical systems, and probability distribution density function $p_q(y)$ is interpreted as the lifetime distribution of the system. All values included in *NSO* get physical sense. It is possible intelligently to choose expressions for $p_q(y)$ (replacing $w(t,t_0)$ from [5, 6]). This interpretation proves to be true by that: 1) the lifetime is equal $y=t-t_0$; 2) the existing finiteness of the lifetime of real systems should take into account by averaging on $p_q(y)$.

The average lifetime of the system $<\Gamma>$ tending to infinity after thermodynamic limiting transition is explained by the fact that the lifetime of infinite system is also infinite. The quasi-equilibrium distribution (2) itself, as well as Gibbs distribution, does not contain lifetime. But the experience shows that all real systems have finite lifetime and are temporary irreversible. It is possible to find an explanation of paradox of reversibility of Newton's dynamics and irreversibility of real complex systems described by this dynamics, for example, in works [5-7, 12-15]. Mathematical operations similar to (8) for the logarithm of distribution (as (1)), or - more generally (for $p_q(y)$):

$$ln\rho(t)=\int_0^\infty p_q(y)ln\rho_q(t-y,-y)dy=ln\rho_q(t,0) - \int_0^\infty (\int p_q(y)dy)(dln\rho_q(t-y,-y)/dy)dy \qquad (9)$$

it is possible to find in [18], [19], where it is shown, how it is possible to construct from random process $\{X(t)\}$ the set of new processes, introducing the randomized operational time. It is supposed that to each value $t>0$ there corresponds the random value $\Gamma(t)$ with the distribution $p^t_q(y)$. The random values $X(\Gamma(t))$ form new random process, which, generally speaking, need not to be of Markovian type any more. In (9) integration by parts in time is carried out at $\int p_q(y)dy_{|y=0}=-1$; $\int p_q(y)dy_{|y\to\infty}=0$; at

$$p_q(y)=\varepsilon exp\{-\varepsilon y\};\ \varepsilon=1/T=<\Gamma>^{-1}, \qquad (10)$$

the expression (9) passes in *NSO* [1,2].

Thus the operations of taking of invariant part [1], averaging on initial conditions [2], temporary coarse-graining [8], choose of the direction of time [5, 21], are replaced by averaging on lifetime distribution. The logarithm of *NSO* (1) is equal the average from the logarithm of quasi-equilibrium distribution (2) on the system lifetime distribution. As in [22] we set some estimation (or management) about values P_j. The task of a estimation or management corresponds to some information on values P_j. Let's assume, that this information consists in assumptions about the finiteness of system lifetime and about exponential distribution $p_q(y)=\varepsilon exp\{-\varepsilon y\}$. We shall note that for the logarithm of nonequilibrium distribution $ln\rho(t)$, given by equality (9), the equation (6) is valid (after replacement $\partial/\partial t$ on $-\partial/\partial y$ and partial integration the rhs of (6) is equal to $dln\rho(t)/dt$). It is true also initial condition $\rho(t_0)=\rho_q(t_0,0)$ [2], if in (9) we assume that $ln\rho(t_0-y,-y)=0$ at $y>0$, as at the moment of time, smaller than t_0, the system does not exist.

Besides the Zubarev's form of NSO [1-3], NSO Green-Mori form [23] is known, where one assumes the auxiliary weight function [5] to be equal $W(t,t`)=1-(t-t`)/t$; $w(t,t`)=dW(t,t`)/dt`=1/t;\ \tau=t-t_0$. After averaging one sets $\tau\to\infty$. This situation at $p_q(u)=w(t,t`)$ coincides with the uniform lifetime distribution. A source in Liouville equation takes the form $J=ln\rho_q/\tau$. In [1] this form of NSO is compared to the Zubarev's form.

It is possible to specify many concrete expressions for lifetime distribution of system, each of which possesses own advantages. To each of these expressions there corresponds own form of a source in Liouville equation for the nonequilibrium statistical operator. Generally for $p_q(y)$ this source looks like

$$J=p_q(0)ln\rho_q(t,\ 0)+\int_0^\infty (\partial p_q(y)/\partial y)(ln\rho_q(t-y,\ -y))dy.$$

Setting the form of the function $p_q(u)$ reflects not only the internal properties of the system, but also the impact of the environment on the open system, its characteristics of the interaction with the environment. In [2] a physical interpretation of the function $p_q(u)$ in the form of the exponential distribution is given as a free evolution of an isolated system governed by the Liouville operator. In addition, the system undergoes random transitions whereas the corresponding representing point in the phase space switches from one phase trajectory to another with exponential probability under the influence of a "thermostat", the random time intervals between consecutive switches growing infinitely. This occurs if the parameter of the exponential distribution tends to infinity after taking thermodynamic limit. But real physical systems are finite-sized. The exponential distribution is suitable for the description of completely random systems. The impact of the environment on a system can have more organized character, for example, for a system in the stationary nonequilibrium state with input and output fluxes; so different can be the interaction between the system and environment, therefore various forms of the function $p_q(u)$ different from the exponential form can be set.

One could name many examples of explicit defining of the function $p_q(u)$. Every definition implies some specific form of the source term J in the Liouville equation, some specific form of the modified Liouville operator and *NSO*. Thus the family of *NSO* is defined. If distribution $p_q(u)$ contains n parameters, it is possible to write down n equations for their expression through the parameters of the system. From other side, they are expressed through the moments of lifetime. There is the problem of optimum choice of function $p_q(u)$ and *NSO*. In [24] to determine the type of function $p_q(u)$ the principle of maximum entropy for the evolution equations with the source is used.

You can make various assumptions about the form of the function $p_q(u)$, while receiving different expressions for the source in the Liouville equation and nonequilibrium system performance.The main difference of this paper from [4, 9-11] and expressions (1), (8), (9) is that the function $p_q(u)$ replaced by the function $p_q(u,\ t)$, as $p^t{}_q(y)$ in [19].

3. Additional terms in the expressions for the fluxes and entropy production

If instead of depending on $p_q(u)$ the dependence of $p_q(u,\ t)$ is considered, it changes the Liouville equation for *NSO* $\rho(t)$. In [2-3] expression for $\Delta\rho(t)=\rho(t)-\rho_q(t)$ is obtained in the form

$$\Delta\rho(t)=-\int_{-\infty}^{t} e^{-\varepsilon(t-t')}U_q(t,t')Q_q(t')iL\rho_q(t')dt' , \tag{11}$$

where $U_q(t,t')=\exp\{-\int_{t'}^{t} Q_q(\tau)d\tau\}$, $Q_q=1-\mathrm{P}_q$ is the operator, additional to the projection operator Kawasaki-Gunton. Effects of the latter on the quantum or classical variable A is defined by

$$\mathrm{P}_q(t)A=\rho_q(t)TrA+\sum_n\{Tr(AP_n)-(TrA)\langle P_n\rangle^t\}\frac{\delta\rho_q(t)}{\delta\langle P_n\rangle^t} ,$$

$Tr(...)$ is the operation of taking the trace [3]. The operation $Tr(...)$ can be interpreted as the integration over the phase space of N particles with subsequent summation over all N [3]. For the case of dependence $p_q(u,t)$ instead of (11) we obtain

$$\Delta\rho(t)=\int_{-\infty}^{t} e^{p_q(0,t)(t'-t)}U_q(t,t')\{-Q_q(t')iL\rho_q(t')+\int_{0}^{\infty}\Delta p_q(u,t')\rho_q(t'-u,-u)du\}dt', \tag{12}$$

where $\Delta p_q(u,t)=p_q(0,t)p_q(u,t)+\frac{\partial p_q(u,t)}{\partial u}+\frac{\partial p_q(u,t)}{\partial t}$. In comparison with [4, 9-11] an additional term $\frac{\partial p_q(u,t)}{\partial t}$ is appears.

We obtain an expression for the fluxes

$$\frac{\partial\langle P_m\rangle^t}{\partial t}=\langle \dot{P}_m\rangle^t{}_q+\sum_n\int_{-\infty}^{t} e^{-p_q(0,t')(t-t')}[\mathrm{M}_{mn}(t,t')F_n(t')+\Delta_m(t,t')]dt', \tag{13}$$

where the first term in square brackets is obtained in [2, 3]

$$\mathrm{M}_{mn}(t,t')=\int_{0}^{1} dx Tr\{I_m(t)U_q(t,t')\rho_q{}^{x}(t')I_n(t')\rho_q{}^{1-x}(t')\}, \tag{14}$$

$I_n(t)=Q(t)\dot{P}_n+(1-P(t))\dot{P}_n$ are dynamic variables of flows, *P(t)* is Mori projection operator acting on the classical and quantum dynamical variables on the rule $P(t)A=\langle A\rangle^t{}_q+\sum_n\frac{\delta\langle A\rangle^t{}_q}{\delta\langle P_n\rangle^t}(P_n-\langle P_n\rangle^t)$, and the second term is a correction to that obtained in [2, 3] expression. The appearance of such an additive caused a general form of the density function of the lifetime distribution. In this case,

$$\Delta_m(t,t')=\int_{0}^{\infty}[p_q(0,t')p_q(u,t')+\frac{\partial p_q(u,t')}{\partial u}+\frac{\partial p_q(u,t')}{\partial t'}]Tr\{I_m(t)U_q(t,t')\rho_q(t'-u,-u)\}du. \tag{15}$$

For $p_q(u)$ in exponential form (10) $\Delta p_q=0$ and, therefore, the addition of (15) is zero. To obtain the expression for entropy production, which also contains an additional term in comparison with the expressions derived in [2, 3]:

$$\frac{dS(t)}{dt}=\sum_{m,n}\int_{-\infty}^{t} e^{-p_q(0,t')(t-t')}F_m(t)[\mathrm{M}_{mn}(t,t')F_n(t')+\Delta_m(t,t')]dt'. \tag{16}$$

4. Estimates of the additional terms

To estimate the value of supplements in terms of flows and entropy production, we use the explicit expression for the function $p_q(u,t)$ obtained in [24] with maximum entropy method. Under certain approximations obtained in [24] expression for the distribution of the lifetime can be written as

$$p_q(u,t)=\frac{p_q(0)e^{-c_i u/F_i}}{1+\frac{p_q(0)}{F_i}e^{-c_i u/F_i}(R(t)-R(t_0))}, \tag{17}$$

$$R(t)=\sum_j\sum_m F_m(t_0)F_j(t_0)\sum_k\frac{<P_kP_jP_m>-<P_jP_m><P_k>}{<P_iP_k>-<P_i><P_k>}+F_i\ln Z(t_0)-$$

$$-\sum_m\sum_j F_j(t_0)\frac{<P_jP_m>-<P_j><P_m>}{<P_iP_m>-<P_i><P_m>}, \tag{18}$$

where we use the rule Zubarev-Peletminsky [1, 5, 25, 26]

$$\vec{w}\vec{\nabla}P_i = \sum_{j=1}^{M} C_{ij}P_j, \quad (i=1,...,M), \quad \frac{d\vec{z}}{dt} = \vec{w}(\vec{z}), \tag{19}$$

where C_{ij} are c-numbers. When considering the local density of the dynamical variables, P_i values may depend on the spatial variables. Then the quantities C_{ij} may also depend on the spatial variables or may be differential operators;

$$C_i = \sum_j C_{ji}F_j(t_0). \tag{20}$$

From the normalization condition we find

$$p_q(0) = F_i(1-e^{-rC_i/F_i^2})/r; \quad r = R(t_0)-R(t).$$

For the distribution of (17) the expression $\Delta p_q(u,t)$, appearing in (15), is equal

$$\Delta p_q(u,t) = p_q(u,t)[p_q(0,t) - \frac{C_i}{F_i} - \frac{p_q(u,t)}{F_i}(\frac{C_i r}{F_i} - \frac{\partial r}{\partial t})].$$

The value $(\frac{C_i}{F_i})^{-1}$ is close to the average lifetime $\langle t-t_0\rangle$, and expression $\frac{C_i r}{F_i} - \frac{\partial r}{\partial t} \approx \frac{r}{\langle t-t_0\rangle} - \frac{\partial r}{\partial t}$. At sharp changes in time value r this value may take a large value.

In the linear approximation in r

$$p_q(0,t_0) = p_q(0) = a = C_i/F_i; \quad p_q(u,t) = ae^{-au}(1+\frac{ar}{F_i}e^{-au}); \quad \Delta p_q = \frac{a^2e^{-2ua}}{F_i}(-ar+\frac{\partial r}{\partial t}).$$

5. Conclusion

In [16-19] the lifetime of the system are considered as functionals of a random process, is the moment to achieve a stochastic process that characterizes the system, a certain threshold, such as zero. This definition is used in the present work. In [11, 27-28] lifetime is included in the range of common physical quantities acting as assessment or management (in terms of information theory) for the quasi-equilibrium statistical operator, provides additional information about the system. Considered in [11, 28] distribution containing lifetime as thermodynamic parameter may be related to the conduct of [4] and in this paper the interpretation NSO, as average over the distribution of the lifetime of the system.

Let's notice, that in a case when value $d\ln\rho_q(t-y, -y)/dy$ (the operator of entropy production σ [1]) in the second item of the right part (9) does not depend from y and is taken out from under integral on y, this second item becomes $\sigma\langle\Gamma\rangle$, and expression (9) does not depend on form of function $p_q(y)$. There is it, for example, at $\rho_q(t)\sim exp\{-\sigma t\}$, $\sigma=cons$. In work [29] such distribution is received from a principle of a maximum of entropy at the set of average values of fluxes.

Form of the density distribution of the lifetime is essential for the kind of expressions for nonequilibrium system performance. A more detailed description $p_q(u)$ compared with the limiting exponential (10) allowing you to describe the real stages of the evolution (and a systems with small lifetimes). Each of the distributions for the lifetime have a certain physical meaning. In queuing theory the various disciplines of service correspond to different expressions for the density distribution of a lifetime. In the stochastic theory of storage by these expressions correspond to different models of the output and input into the system.

It is shown that the record of dependence of this function on the current point in time leads to additional terms in the expressions for the average flows, of entropy production and other characteristics of a nonequilibrium system.

If type of source in Liouville equation for a non-equilibrium statistical operator in the form of Zubarev [2] it is possible to compare with a linear relaxation source in Boltzmann equation, more difficult types of sources, got from other distributions for lifetime of the system, it is possible to compare to more realistic type of integral of collisions, that is explained by the openness of the system, by its co-operation with surroundings and finiteness of lifetime of the system, and also coarsening for physically infinitely small volumes.

In [30] it was noted that the role of the form of the source term in the Liouville equation in *NSO* method has never been investigated. In [19] it is stated that the exponential distribution is the only one which possesses the Markovian property of the absence of contagion*,* that is whatever is the actual age of a system, the remaining time does not depend on the past and has the same distribution as the lifetime itself.

The physical sense of averaging on entered lifetime distribution of quasi-equilibrium system as it was already marked, consists in the obvious account of infringement of time symmetry and loss (reduction accessible) the information connected with this infringement, that is shown in occurrence the value of average of entropy production $<\Delta S\ (t)>$ not equal to zero, obviously reflecting fluctuation-dissipative processes at the real irreversible phenomena in non-equilibrium systems. The correlations received in the present paper generalize results of statistical non-equilibrium thermodynamics [1, 2, 3] and information statistical thermodynamics [4-5] as instead of weight function of a form $\varepsilon exp\{\varepsilon t'\}$ contain density of probability of lifetime of quasi-equilibrium system which as it was already marked, can not coincide with exponential distribution (in the latter case it coincides with weight function from [1, 2, 3]). For example, for system with n classes of ergodic states limiting exponential distribution is replaced with the general Erlang. In research of lifetimes of complex systems it is possible to involve many results of the theory of reliability, the theory of queues, the stochastic theory of storage processes, theory of Markov renewal, the theory of semi-Markov processes etc.

As it is specified in [31], existence of time scales and a stream of the information from slow degrees of freedom to fast create irreversibility of the macroscopical description. The information continuously passes from slow to fast degrees of freedom. This stream of the information leads to irreversibility. The information thus is not lost, and passes in the form inaccessible to research on Markovian level of the description. For example, for the rarefied gas the information is transferred from one-partial observables to multipartial correlations. In work [4] values $\varepsilon = 1\ /\ <\Gamma>$ and $p_q\ (u)\ =\ \varepsilon exp\ \{-\varepsilon u\}$ are expressed through the operator of entropy production and, according to results [31], - through a stream of the information from relevant to irrelevant degrees of freedom.

Introduction in *NSO* to function $p_q(u)$ corresponds to specification of the description by means of the effective account of communication with irrelevant degrees of freedom. In the present work it is shown, how it is possible to spend specification the description of effects of memory within the limits of method *NSO*, more detailed account of influence on evolution of system of quickly varying variables through the specified and expanded kind of density of function of distribution of time the lived system of a life.

In many physical problems finiteness of lifetime can be neglected. Then $\varepsilon \sim 1/<\Gamma> \rightarrow 0$. For example, for a case of evaporation of drops of a liquid it is possible to show [32], that non-equilibrium characteristics depend from $exp\{y^2\}$; $y=\varepsilon/(2\lambda_2)^{1/2}$, λ_2 is the second moment of correlation function of the fluxes averaged on quasi-equilibrium distribution. Estimations show, what even at the minimum values of lifetime of drops (generally - finite size) and the maximum values ε size $y=\varepsilon/(2\lambda_2)^{1/2} \leq 10^{-5}$. Therefore finiteness of values $<\Gamma>$ and ε does not influence on behaviour of system and it is possible to consider $\varepsilon = 0$. However in some situations it is necessary to consider finiteness of lifetime $<\Gamma>$ and values $\varepsilon > 0$. For example, for nanodrops already it is necessary to consider effect of finiteness of their lifetime. For lifetime of neutrons in a nuclear reactor in work [4] the equation for $\varepsilon = 1/<\Gamma>$ which decision leads to expression for average lifetime of neutrons which coincides with the so-called period of a reactor is received. We have in work [33] account of finiteness of lifetime of neutrons result to correct distribution of neutrons energy.

References

[1] D. N. Zubarev, *Nonequilibrium statistical thermodynamics* (Plenum-Consultants Bureau, New York, 1974).

[2] D. N. Zubarev, in *Reviews of Science and Technology: Modern Problems of Mathematics.* Vol.15, pp. 131-226, (in Russian) ed. by R. B. Gamkrelidze, (Izd. Nauka, Moscow, 1980) [English Transl.: J. Soviet Math. ***16***, 1509 (1981)].

[3] D. N. Zubarev, V. Morozov, G. Ropke, *Statistical Mechanics of Nonequilibrium Processes: Basic Concepts, Kinetic Theory* (John Wiley & Sons, New York, 1996).

[4] V. V. Ryazanov, Fortschritte der Phusik/Progress of Physics **49**:8-9, 885 (2001).

[5] J. G. Ramos, A. R. Vasconcellos and R. Luzzi, Fortschr. Phys./Progr. Phys. **43**:4, 265 (1995).

[6] J. G. Ramos, A. R. Vasconcellos and R. Luzzi, Fortschr. Phys./Progr. Phys. **47**:6, 937 (1999).

[7] I. Prigogine, *From Being to Becoming* (Freeman, San Francisco, 1980).

[8] J. G. Kirkwood, J.Chem.Phys. **14**:1, 180 (1946); **15**:1, 72 (1946).

[9] V. V. Ryazanov, Fizika nizkikh temperature **33**:9, 1049 (2007).

[10] V. V. Ryazanov, International Journal of Theoretical and Mathematical Physics **2**:1 1 (2012).

[11] V. V. Ryazanov, European Physical Journal B **72**: 4, 629 (2009).

[12] J. R. Dorfman, P. Gaspard, Phys.Rev.E **51**:1, 28 (1995).

[13] P. Gaspard, Chaos **3**:4, 427 (1993).

[14] P. Gaspard, J.R. Dorfman, Phys.Rev.E **52**:4, 3525 (1995).

[15] P. Gaspard, *Physica A* **263**:1, 315 (1999).

[16] V. S. Korolyuk and A. F. Turbin, Mathematical Foundations of the State Lumping of Large Systems (Kluwer Acad.Publ., Dordrecht, Boston/London, 1993).

[17] R. L. Stratonovich, *The elected questions of the fluctuations theory in a radio engineering* (Gordon and Breach, New York, 1967).

[18] D. R. Cox, *Renewal theory* (London: Methuen; New York: John Wiley, 1961).

[19] W. Feller, *An Introduction to Probability Theory and its Applications*, vol.2 (J.Wiley, New York, 1971).

[20] D. R. Cox and D. Oakes, *Analysis of Survival Data* (Chapman and Hall, London, New York, 1984).

[21] A. R. Vasconcellos, R. Luzzi, L. S. Garsia-Colin, Phys. Rev. A **43**:12 6622 (1991).

[22] R. L. Stratonovich, I*nformation theory* (in Russian) (Izd. “Sovetskoe Radio”, Moscow, 1966).

[23] H. Mori, I. Oppenheim and J. Ross, *Some Topics in Quantum Statistics, in Studies in Statistical Mechanics*, I. Ed. by J. de Boer and G. E. Uhlenbeck (North-Holland, Amsterdam, 1962).

[24] V. V. Ryazanov, Maximum entropy principle and the form of source in non-equilibrium statistical operator method. Preprint. Cond-mat, arXiv:0910.4490v1.

[25] S. V. Peletminskii, A. A. Yatsenko, Soviet Phys JETP **26** 773 (1968); Zh. Eksp. Teor. Fiz. **53**, 1327 (1967).

[26] A. I. Akhiezer, S. V. Peletminskii, *Methods of statistical physics* (Oxford: Pergamon, 1981)

[27] V. V. Ryazanov, The weighed average geodetic of distributions of probabilities in the statistical physics. Preprint. Cond-mat, arXiv:0710.1764.

[28] V. V. Ryazanov, S. G. Shpyrko, Condensed Matter Physics **9**:1(45), 71 (2006).

[29] R. Dewar, J. Phys.A: Math. Gen., **36**:3 631 (2003).

[30] V. G. Morozov, G. Röpke, Condensed Matter Physics **1**:4(16), 673 (1998).

[31] J. Rau, B. Muller, Physics Reports **272**, 57 (1996) .

[32] V. V. Ryazanov, Colloid Journal **68**:2, 217 (2006).

[33] V. V. Ryazanov, Atomic Energy 99:5, 782 (2005).